Quasi-Guided Modes and Related Radiation Losses in Optical Dielectric Waveguides with External Higher Index Surroundings

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Abstract—**Mode filter actions are found theoretically in an optical dielectric waveguide consisting of a core and a thin cladding layer which is further surrounded by an external higher index region. The propagating waves, which are usually considered to be cutoff modes, can be guided with a small amount of loss under a certain condition. These waves are defined here as quasi-guided modes. These modes tend to the guided modes of the guide when the cladding thickness increases infinitely.**

A method is given to estimate the losses. As an example, the radiation losses are formulated for a symmetric slab waveguide, and are found to be approximately proportional to the cube of the mode number of the quasi-guided mode. Therefore, losses of the quasi-guided modes depend strongly on the mode number.

It is suggested that fibers with large core diameters can be used as quasi-single mode fibers by covering the clad-type multimode fibers with external higher index surroundings and choosing the parameters properly.

I. INTRODUCTION

MULTIMODE dielectric waveguides such as multimode optical fibers are very attractive for the transmission of optical signals because of their larger core dimension. Therefore, these structures offer improved handling properties together with smaller scattering loss due to the irregularities of the boundaries which is inversely proportional to the third power of the core dimension [1]. On the other hand, in a multimode fiber, the group velocity dispersion limits the bandwidth and, therefore, it is very important to find a mode-dependent filtering mechanism in this type of multimode dielectric waveguide.

A transverse index distribution in which the refractive index of the external layer is higher than the minimum value of the index at the interior region, seems to be one of the useful types of index distribution for the transmission medium, in optical communication systems and integrated optics, because of its mode-dependent radiation properties. In relation to the index distribution mentioned above, the prism coupler has been treated, using the plane wave expansion method. However, the main purpose was the treatment of excitation problems [2], [3]. Generally speaking, the propagation and radiation properties, and especially the mode-dependency of the radiation losses, are not yet fully known.

This paper is concerned with the propagating modes and the radiation losses of dielectric waveguides which

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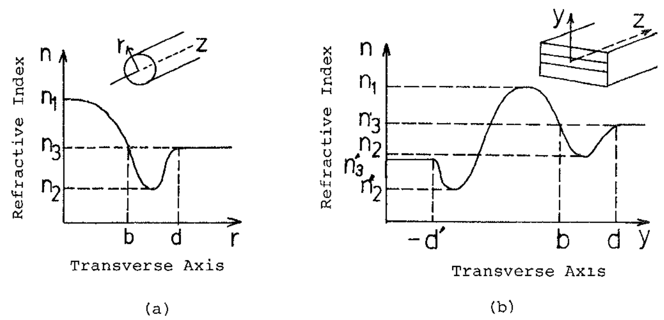

Fig. 1. Transverse refractive index distributions with external higher index surroundings. (a) Axial symmetric guide. (b) Slab guide.

consist of a core and a thin cladding with external higher index surroundings. Wave propagations along the dielectric waveguide is analyzed with the aid of expansions in terms of characteristic modes. Quasi-guided modes (QGM's) are defined to be linear combinations of the radiation modes with real continuous propagation constants at finite regions on the spectrum. These QGM'S correspond to propagating modes with attenuation due to the radiations toward external surroundings. These modes tend to the attenuationless guided modes of the guide when the cladding thickness approaches infinity. As an example of this QGM analysis, clad-type slab dielectric waveguides with external higher index surroundings are treated numerically and analytically. As a result of the analysis, mode-dependency of the radiation loss is represented quantitatively. Finally, it is suggested that fibers of large-diameter core can be used as quasi-single mode fibers by covering clad-type multimode fibers with external higher index surroundings.

II. QUASI-GUIDED MODES

Wave propagation along cylindrical dielectric waveguides with transverse refractive index distributions, as shown in Fig. 1, is considered. It would be possible to approximate the above-mentioned index distribution by an external higher index surrounding of finite thickness under the following conditions. The thickness of the surrounding is assumed to be sufficiently large compared to the core width, and the outermost surface of the surrounding is either rough or absorptive, to prevent reflections of the radiated light. With the aid of the characteristic mode expansion, one of the field component of an electromagnetic wave $\psi(r,z)$ is represented as

$$\psi(r,z) = \sum \int L(\beta)N(\beta)^2F(r;\beta) \exp(-j\beta z)\, d\beta \qquad (1)$$

where z is the wave propagation axis and r is a position vector in a transverse cross section of the guide. $\sum\int$ implies both summation and integration over the whole spectrum of propagation constants β's of the real value. It is assumed that a spatial variation in the transverse direction of an input field $\psi(r,0)$ is slow enough to be expanded with only modes of real propagation constants. But this assumption is considered to be good approximation for practical cases. $N(\beta)$ is a normalization function defined with respect to the time-averaged Poynting power,

$$N(\beta)^2 \int_c M(r;\beta)F(r;\beta)F(r;\beta')\,dr \tag{2}$$
$$= \delta(\beta - \beta'), \qquad \text{for continuous spectrum}$$

where the integration extends over the entire cross section c. $N(\beta)$ can be regarded as the normalized amplitude of the field in the core region for the radiation modes (see Appendix II). M is a factor that makes the product $(N(\beta)F(r;\beta))\cdot(M(r;\beta)N(\beta)F(r;\beta))$ a Poynting power density. An example of this factor will be shown in Section III. In the case of a discrete spectrum, $N(\beta)$, $M(r;\beta)$, $F(r;\beta)$, $F(r;\beta')$, and $\delta(\beta-\beta')$ are to be replaced by N_r, $M_r(r)$, $F_r(r)$, $F_g(r)$, and δ_{rg}, respectively. Degenerate modes are also required to be orthogonal. Then excitation factors $L(\beta)$ (or L_r) are determined by input-field distributions $\psi(r;0)$, using (1) and (2), as

$$L(\beta) = \int_c \psi(r;0)M(r;\beta)F(r;\beta)\,dr. \tag{3}$$

In the case of dielectric waveguides with external higher index surroundings as shown in Fig. 1, normalization functions $N(\beta)$ have sharp peaks along the β-axis, in the range of $k_2 < \beta < k_3 (k_i = (2\pi/\lambda)n_i$; n_i is the refractive index). Therefore, in this range the parts of β-spectrum around these peaks are the dominant contribution to the integration in (1), and since $N(\beta)$ varies rapidly, $L(\beta)$ and $F(r;\beta)$ can be taken out of the integration. Then (1) may be represented approximately as follows to give the field around the core of the guide:

$$\psi(r;z) \simeq \sum_r L_r N_r \Phi_r(r,z) + \sum_p L(\beta_p)\hat{\Phi}_p(r,z)$$
$$(k_3 < \beta < k_1) \qquad (k_2 < \beta < k_3)$$
$$+ \sum \int_0^{k_2} L(\beta)N(\beta)\Phi(r,z;\beta)\,d\beta \tag{4}$$

where Φ_r and $\Phi(\beta)$ are normalized guided and radiation modes, respectively, having the form of $N(\beta)F(r;\beta)\exp(-j\beta z)$. $\sum_p$ implies the summation of all peaks mentioned above, and $\hat{\Phi}_p$ is given by

$$\hat{\Phi}_p(r,z) \equiv F(r;\beta_p)\int_{\beta_p-\delta_p}^{\beta_p+\delta_p} N(\beta)^2 \exp(-j\beta z)\,d\beta. \tag{5}$$

Once δ_p is sufficiently large compared with the width of the peak, the integrated value is almost independent of the value δ_p. β_p is the propagation constant which gives the pth peak of $N(\beta)$ on the β-axis. It is assumed that the width of the peak is sufficiently small compared with β_p.

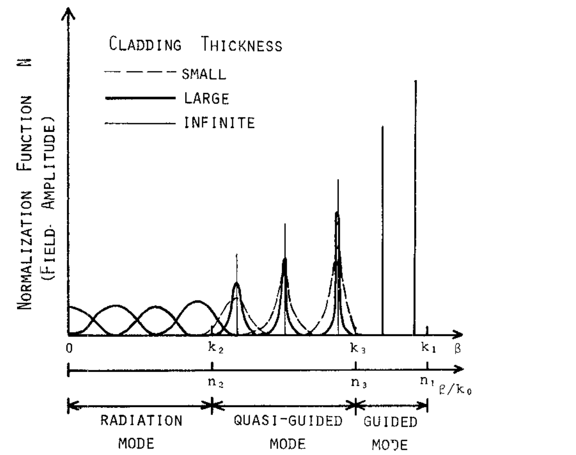

Fig. 2. Conceptual scheme for the transition of normalization function from quasi-guided to guided mode. By increasing the cladding thickness, the peaks will become sharper.

The propagation properties of $\hat{\Phi}_p$ are determined almost solely by the peak-widths of the normalization function $N(\beta)$, which are inherent in the waveguide under consideration. Assuming that β_p' is a propagation constant near β_p, then the field $\hat{\Phi}_p$ is represented as follows (Appendix I):

$$\hat{\Phi}_p \cong F(r;\beta_p')\pi\{ff''/2 - f'^2/4\}^{-1/2}$$
$$\cdot\exp(-\alpha_p z)\exp(-j\hat{\beta}_p z), \qquad \text{(around core)} \tag{6}$$

where

$$\alpha_p = [(2f - f'^2/f'')/f'']^{1/2} \tag{7}$$

$$\hat{\beta}_p = \beta_p' - f'/f'' \tag{8}$$

and

$$f \equiv 1/N(\beta)^2. \tag{9}$$

f, f', and f'' designate the value of the function defined by (9), and its first and second derivatives at β_p', respectively. Unless β_p' is too far from β_p, the propagation constant $\hat{\beta}_p$ and decay constant α_p are almost independent of β_p', and $\hat{\beta}_p \simeq \beta_p$. The field $\hat{\Phi}_p$ propagates keeping its transverse field distribution close to the core, but decaying exponentially along the z-axis. From (7) and (9), the following equation may be derived to estimate approximately the decay constants from peak-widths of normalization function:

$$N(\beta_p \pm \alpha_p) \simeq N(\beta_p)/\sqrt{2}. \tag{10}$$

The decay constant α_p is approximately equal to half of the width at points corresponding to $1/\sqrt{2}$-values of the peak of the normalization function N. Then scanning of β along the real axis will enable us to determine α_p from the width of the peak of $N(\beta)$. So this method is easily applicable to the guide of very complicated refractive-index distribution as shown in Fig. 1(b).

Increasing the cladding thickness $(d-b)$ infinitely, the positions of the peaks of the normalization function tend to correspond to the propagation constants of the guided modes of the guide without external higher index surroundings and the peak-widths, and, therefore, decay constants will shrink to zero, as shown in Fig. 2. That is, the field $\hat{\Phi}_p$ tends to become the guided mode in the limit of $(d-b)$

$\to \infty$. An example of this transition to the guided mode will be shown numerically in Section III.

The field $\hat{\Phi}_p$ is a linear combination of radiation modes around one of the peaks of the normalization function N, and forms one "group" propagating with a small amount of loss. The field at far distances from the core in the transverse direction can be calculated by (5). However, in this case, due to the rapid variation of $F(r;\beta)$ with β, it should be returned back into the integration sign. Therefore, the fields of QGM's vanish at the distances far from the core, as shown in Fig. 5. It forms a quasi-mode which is independent of the manner of excitation by the input wave. This loss is dependent on the waveguide parameters and the peak number. Hence we define the field $\hat{\Phi}_p$ as the QGM.[1] In investigating wave propagations near the core, it is effective to consider these new modes QGM.

III. QUASI-GUIDED MODES IN SLAB WAVEGUIDES

As an example of the analysis developed in the previous section, QGM's in waveguides with transverse index distributions (shown in Fig. 3) are analyzed. Characteristic modes corresponding to the spectrum range of $n_2 < \beta/k_0 < n_3$ $(k_0 = 2\pi/\lambda)$ are derived from wave equations. In slab waveguides, the factor M is $\beta/(2\omega\mu)$ for TE, and $\beta n_1^2/(2\omega\mu n^2(y))$ for TM waves, where μ stands for permeability. Then the normalization function $N(\beta)$ is given as follows (Appendix II):

$$N(\beta) = \left[\frac{\pi\gamma_3}{2\omega\mu\zeta_3}\left\{(AU + BU^{-1})^2 + \left(\frac{\zeta_3\kappa}{\zeta_2\gamma_3}\right)^2 (AU - BU^{-1})^2\right\}\right]^{-1/2} \quad (11)$$

where

$$U = \exp\{\kappa(d-b)\} \quad (12)$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \left\{ \cos(\gamma_1 b - \phi) \begin{pmatrix} - \\ + \end{pmatrix} \frac{\gamma_1\zeta_2}{\kappa} \sin(\gamma_1 b - \phi) \right\} \Big/ 2 \quad (13)$$

$$\gamma_i = (k_i^2 - \beta^2)^{1/2}$$

$$\kappa = (\beta^2 - k_2^2)^{1/2}$$

$$\zeta_1 = \begin{cases} 1, & \text{for TE waves} \\ (n_i/n_1)^2, & \text{for TM waves} \end{cases} \quad (14)$$

[1] A QGM as defined here is close to the leaky wave [4]. The half-width at half-height of the peak of N^2, which gives the decay constant due to the radiation loss, is identical to the imaginary part of the corresponding leaky wave pole [3]. But the field of the leaky wave increases to infinity with the transverse distance, and therefore it cannot be excited in its pure form [5]. As for QGM, the field vanishes with the transverse distance, so that this mode itself can be considered as a wave carrying finite power.

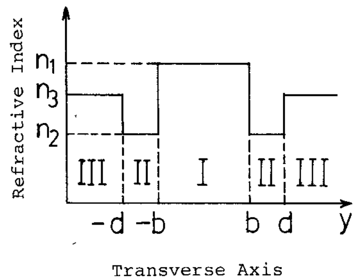

Fig. 3. Refractive index distribution of symmetric slab waveguide.

where $i = 1,2,3$ and ϕ is 0 for even modes, and $\pi/2$ for odd modes. In the case of $U \gg 1$, it is shown by (11) that the normalization function $N(\beta)$ has peaks on the β-axis around propagation constants which make A equal to zero. For finite values of U, the width of the peak is finite and the propagation constant $\hat{\beta}_p$ of a QGM is given as the position of the pth peak of $N(\beta)$. This position is slightly different from the root β_p' of $A = 0$, which gives the propagation constant of the guided mode for a guide without an external surrounding. In the limit of $U \to \infty$ $(d \to \infty)$, the propagation constants of QGM $\hat{\beta}_p$ tend to coincide with the corresponding propagation constant of guided mode of the guide with cladding of infinite thickness. In this case the peak-width and therefore the decay constant of QGM tend to be zero.

Numerical examples are shown in the following figures. Fig. 4 shows the normalization function $N(\beta)$ for the waveguides with parameters of film thickness $2b = 5\lambda/n_1$, normalized refractive indexes $n_2/n_1 = 0.99$, $n_3/n_1 = 0.997$, and TE_1 mode operation. If the cladding thickness $(d - b)$ is increased, the peak becomes sharper. At $d = 10\lambda/n_1$, the excitation factor $L(\beta)$, which is calculated by substituting the field distribution of the TE_1 mode of the guide with cladding of infinite thickness $(d = \infty)$ into $\psi(y,0)$, is a sufficiently slowly varying function compared with $N(\beta)$. Fig. 5 shows the transverse field amplitude distributions at various distances z, for the case of $d = 10\lambda/n_1$, where E_x corresponds to Φ_x as mentioned in Appendix II. These distributions are calculated numerically based on (1). It is shown that the distribution of field amplitude associated with QGM which localizes around the core at $z = 0$, diverges over wider transverse cross sections as propagating down the waveguide. Far from the core, for example, at $y = 1000\lambda/n_1$, the field becomes appreciable for $z > 4000\lambda/n_1$, and it is found from the calculation of the phase-distributions of QGM that far from the core, the wave propagates outward as a plane wave whose normal inclines by the angle $\cos^{-1}(\beta/k_3)$. It is also confirmed that the field amplitude decays exponentially with distance z. The exponential decay constant coincides with that obtained from (10) and Fig. 4. Therefore, the QGM analysis seems to be sufficiently precise in the case of $d = 10\lambda/n_1$. Furthermore, exponential decays are confirmed numerically up to $d = 6\lambda/n_1$, or $L = 1.1$ dB/$(100\lambda/n_1)$. For values of d smaller than $6\lambda/n_1$, the decay will be different from exponential type in our numerical example.

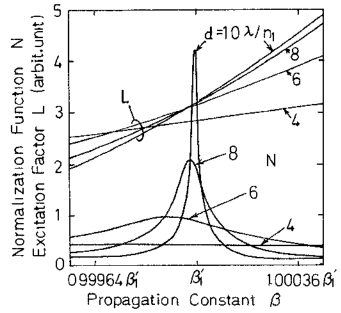

Fig. 4. Normalization function N and excitation factor L. The half-width at the $1/\sqrt{2}$ points with respect to the maximum gives the decay constant.

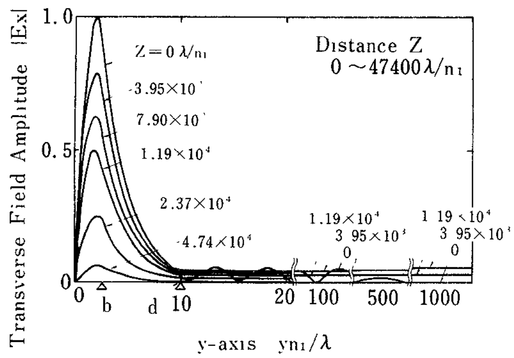

Fig. 5. Propagation properties of quasi-guided mode. $d = 10\lambda/n_1$.

For slab waveguides with general index distributions [6] shown in Fig. 1(b), QGM-analysis will be applied directly using the normalization function $N(\beta)$ given by

$$N(\beta) = \left[\frac{\pi}{4\omega\mu}\left[\frac{\gamma_l}{\zeta_l}\left\{\Phi_x^2(-d';\beta) + \left(\frac{k_1\zeta_l}{\gamma_l}\right)^2 \Phi_z^2(-d';\beta)\right\}^2 + \frac{\gamma_r}{\zeta_r}\left\{\Phi_x^2(d;\beta) + \left(\frac{k_1\zeta_r}{\gamma_r}\right)^2 \Phi_z^2(d;\beta)\right\}^2\right]\right]^{-1/2} \tag{15}$$

where Φ_x and Φ_z are x and z components of fields, respectively, and

$$\Phi_z = -\frac{1}{k_1\zeta}\frac{\partial}{\partial y}\Phi_x \tag{16}$$

where subscripts l and r imply regions of $y \leqq -d'$ and $y \geqq d$, respectively. Relations between fields $\Phi_x(-d')$, $\Phi_z(-d')$, $\Phi_x(d)$, and $\Phi_z(d)$ can be established, for example, using transverse F-matrix method [7]. Furthermore, QGM's of waveguides with circular cross section, shown in Fig. 1(a), can be analyzed similarly.

IV. RADIATION LOSSES OF QUASI-GUIDED MODES IN SYMMETRICAL SLAB WAVEGUIDES

Radiation loss formulas which relate decay constants with waveguide parameters will be derived for refractive index distributions shown in Fig. 3. Using (7), (9), and (11), and assuming the following approximation condition

$$U = \exp\{\kappa(d-b)\} \gg 1 \tag{17}$$

the power loss is represented as follows:

$$\begin{aligned} L_p(\mathrm{dB/m}) &= 8.7\alpha_p \\ &\simeq 35\frac{\gamma_3'}{k_1\zeta_3}\left[\left\{1+\left(\frac{\zeta_2\gamma_3'}{\zeta_3\kappa'}\right)^2\right\}\left\{1+\left(\frac{\kappa'}{\zeta_2\gamma_1'}\right)^2\right\}\right. \\ &\quad \left.\cdot\left\{b+\frac{1}{\zeta_2\kappa'}\left(1+\left(\frac{\kappa'}{\gamma_1'}\right)^2\right)\Big/\left(1+\left(\frac{\kappa'}{\zeta_2\gamma_1'}\right)^2\right)\right\}\right]^{-1} \\ &\quad \cdot\exp\{-2\kappa'(d-b)\}. \end{aligned} \tag{18}$$

(approximation formula A for TE and TM modes) where γ_1', κ', and γ_3' are transverse propagation constants in the three regions of the waveguide, given by (14) with β being the root β_p' of the equation $A = 0$. In order to use (18), it is necessary to calculate β_p'. However, for TE modes far from cutoff, losses can be calculated directly from waveguide parameters, with the following approximation formula:

$$L_p(\mathrm{dB/m}) \simeq \frac{\Delta}{\lambda/n_1} R(T,p,h,g) \tag{19}$$

(approximation formula B for TE modes far from cutoff) where p is the mode number in a system where guided modes are counted from 0 followed by the QGM's up to m, so that the propagation constant β_p assumes smaller values for higher p's:

$$R = 440\frac{x^2(1-x^2)^{3/2}(h-1+x^2)^{1/2}}{h[T(1-x^2)^{1/2}+1]} \cdot\exp[-2gT(1-x^2)^{1/2}] \tag{20}$$

where

$$\Delta \equiv \frac{n_1^2-n_2^2}{2n_1^2} \simeq \frac{n_1-n_2}{n_1} : \text{normalized index difference;}$$

$$T \equiv (2\Delta)^{1/2}k_1b \simeq \frac{\pi}{2}(m+1) : \text{normalized core width;}$$

$$h \equiv \frac{n_3^2-n_2^2}{n_1^2-n_2^2} \simeq \frac{n_3-n_2}{n_1-n_2} : \text{external index constant;}$$

$$g \equiv \frac{d-b}{b} : \text{normalized cladding thickness;}$$

$$x \equiv \frac{\pi}{2}\frac{p+1}{T} \simeq \frac{p+1}{m+1}. \tag{21}$$

In Fig. 6, radiation losses calculated from approximation formulas A and B are compared to those from numerical calculations of N. It is shown that approximation formula A can be applied when there exists one higher order mode in addition to the QGM under consideration, while formula B can be applied if there are two higher order QGM's than that of the considered one. The left ordinate gives the value of R, with which radiation losses can be calculated for general parameters, using (19). The

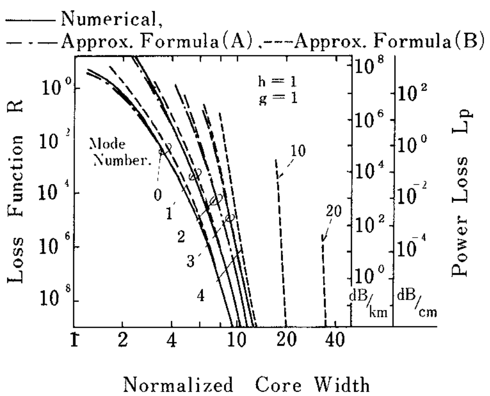

Fig. 6. Loss versus normalized core width. Approximation formulas *A* and *B* coincide with the numerical results if there exist, respectively, one and two higher modes in addition to the QGM under consideration.

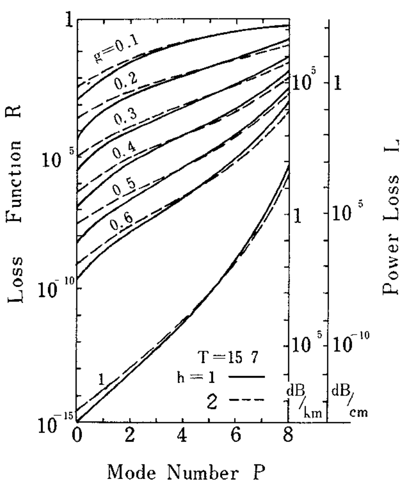

Fig. 7. Mode-dependency of radiation loss. The solid lines are for the case of $h = 1$, while the dotted lines are for $h = 2$.

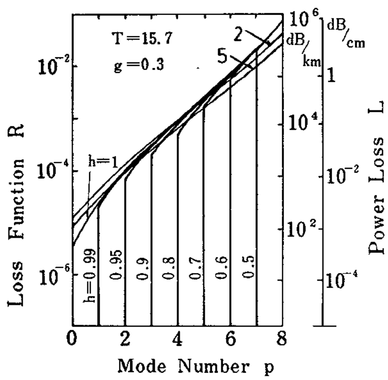

Fig. 8. Mode-dependency of radiation loss.

right ordinates of Figs. 6–8 give the numerical examples corresponding to the following values of parameters: $\lambda = 1\ \mu m$, $n_1 = 1.5$, $\Delta = 1$ percent.

Fig. 7 shows mode-dependencies of radiation losses in the case of $h = 1$ and 2. In the case of $h = 1$, the indexes of the core and the external surrounding are the same. As an example, at $g = 0.4$, for TE_0, TE_1, and TE_8. modes, radiation losses are 2, 16, 3×10^5 dB/km, respectively. Especially at lower order modes, radiation loss formula is represented approximately for $h = 1$, $T \gg 1$, $x^2 \ll 1$ as follows:

$$L_p(\text{dB/m}) = 280 \frac{\Delta}{m^4\lambda/n_1} \exp\{-2gT\}(p+1)^3. \quad (22)$$

As shown in Fig. 8, in the case that $h < 1$, i.e., index n_3 of the external surrounding is lower than that of the core, no loss transmission, up to a desired order mode and high losses for the remaining higher order modes, is possible by adjusting the value of h. On the other hand, in the case of $h > 1$, the loss is not so sensitive to the index n_3 of the external surrounding.

As the normalized refractive index difference Δ is small compared to unity, the previously mentioned numerical results may be applied also to TM modes.

Radiation losses in a waveguide which has a rectangular cross section core of dimensions $2a$ and $2b$ along the x and y axes, respectively, can be estimated approximately by separating eigenfunctions into two functions of transverse axes x and y. In this case, the losses are represented as the summation of α_p^x, and α_q^y, where for example, α_p^x is the pth QGM loss of the slab waveguide which is given by $b \to \infty$ and, therefore, can be calculated as mentioned earlier.

Because of strong mode dependencies of the loss, the multimode fibers can be used as fibers which carry only a few lower order modes. Therefore, bandwidths are broadened by covering clad fibers with, external higher index surrounding. Besides the properties mentioned above, external higher index surrounding optical fibers are considered to have the following merits, due to their large core diameters; low scattering losses caused by boundary irregularities, large light acceptance areas together with improved connection losses. But the light acceptance angle becomes critical. The external higher index surrounding optical fibers may be used as homogeneous, and lump-loaded lines.

Silica-core fibers with outer jackets of fused silica which serve to strengthen and protect the waveguide have been reported previously [8]. For example, by reducing the cladding thickness of waveguides to desired values, the external higher index surrounding optical fibers mentioned above can be realized.

V. CONCLUSION

Propagation modes in dielectric waveguides with external higher index surrounding are formulated. The quasi-guided modes which propagate with radiation decay are defined as linear combinations of real continuous radiation modes.

Radiation loss properties of clad-slab waveguides are analyzed, and it is revealed that remarkable mode-dependency exists in this loss mechanism. It is found that because of such mode-dependencies of losses, multimode fibers can be used effectively as single mode fibers and therefore with broadened bandwidths by covering clad fibers with external higher index surroundings.

APPENDIX I

Equation (5) is calculated as

$$\int_{\beta_p'-\delta_p}^{\beta_p'+\delta_p} N(\beta)^2 \exp(-j\beta z)\, d\beta$$

$$\simeq \int \frac{\exp[-j(\beta-\beta_p')z]\exp[-j\beta_p'z]}{f+f'(\beta-\beta_p')+f''(\beta-\beta_p')^2/2}\, d\beta$$

$$\simeq \{2\exp[-j(\beta_p'-f'/f'')z]\}/f''$$

$$\cdot \int_{-\infty}^{\infty} \frac{\exp(-jvz)}{v^2+(2f-f'^2/f'')/f''}\, dv \qquad (23)$$

where the arguments of f, f', and f'' are β_p'. Equations (6)–(8) are obtained, if the above integration is performed.

APPENDIX II

A transverse field component Φ_x is represented as follows: in the braces { }, the upper line is applied to TE waves, while the lower line is applied to TM waves:

$$\Phi_x(y,z;\beta) = \begin{Bmatrix} E_x \\ (1/n_1)(\mu/\epsilon_0)^{1/2}H_x \end{Bmatrix} = N(\beta)F(y;\beta)\exp(-j\beta z). \qquad (24)$$

Substituting (24) into the wave equation, the eigenvalue equation for $F(y;\beta)$ is derived. The eigenfunctions $F(y;\beta)$, whose eigenvalues belong to the range of $k_0^2n_2^2 < \beta^2 < k_0^2n_3^2$, are given as follows. As the refractive index distribution is symmetric, only the region of $y \geqq 0$ is considered:

$$F(y;\beta) = \begin{cases} \cos(\gamma_1 y - \phi), & \text{I. } 0 \leqq y \leqq b \\ A\exp\{\kappa(y-b)\} + B\exp\{-\kappa(y-b)\}, & \text{II. } b \leqq y \leqq d \\ C\sin\{\gamma_3(y-d)+\psi\}, & \text{III. } y \leqq d \end{cases} . \qquad (25)$$

From the boundary conditions,

$$C = \left\{(AU+BU^{-1})^2 + \left(\frac{\kappa\zeta_3}{\gamma_3\zeta_2}\right)^2 (AU-BU^{-1})^2\right\}^{1/2} \qquad (26)$$

$$\psi(\beta) = \tan^{-1}\left(\frac{\gamma_3\zeta_2}{\kappa\zeta_3}\frac{AU+BU^{-1}}{AU-BU^{-1}}\right) \qquad (27)$$

and other parameters are defined in (12)–(14). Using the wave equation and the continuities of F and $(1/\zeta)\cdot(\partial F/\partial y)$ which are proportional, respectively, to x and z components of the field at the boundaries, the integral in (2) is calculated as follows:

$$\int_{-\infty}^{\infty} \frac{\beta}{2\omega\mu\zeta} F(y;\beta)F(y;\beta')\, dy = \frac{1}{2\omega\mu\zeta_3}\frac{1}{\beta^2-\beta'^2}$$

$$\cdot\left[\frac{\partial F(y;\beta)}{\partial y}F(y;\beta') - F(y;\beta)\frac{\partial F(y;\beta')}{\partial y}\right]_{-\infty}^{\infty}. \qquad (28)$$

Substituting (25) into (28), the right-hand side of (28) is

$$(\pi\gamma_3/2\omega\mu\zeta_3)C^2\delta(\beta-\beta'). \qquad (29)$$

From (2), (29), and (26), (11) is obtained.

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